Reply to "Comment on 'Scaling behavior in explosive fragmentation' "

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Following the suggestion in the Comment on our previous work by Åström, Linna, and Timonen [Phys. Rev. E **65**, 048101 (2002)], we performed extensive molecular-dynamics simulations to confirm that our numerical results for the mass distribution of fragments after the "explosion" of thermalized samples are consistent with the scaling form $n(m) \sim m^{-(\alpha+1)} f(m/M_0)$, where $f(m/M_0)$ is a cutoff function, M_0 is a cutoff parameter, and the exponent α is close to zero.

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The authors of the preceding Comment [1] argue that an erroneous interpretation of the fragmentation process can be made if the scaling analysis is performed in terms of the quantity $N(m)=F(m)/m$, where $F(m)$ is the cumulative mass distribution,

$$F(m)=\int_m^\infty n(m')dm'. \tag{1}$$

We certainly agree with this point. Indeed, if one assumes that $F(m) \sim m^{-\alpha}$, it follows that the $1/m$ factor in $N(m)$ can surely induce a numerical artifact in the region of very small values of m, if the exponent α is close to zero. In order to clarify this issue, we carried out additional molecular-dynamics simulations for 50 independent realizations of the many-particle system, thermalized at two different temperatures. After thermalization, each object was fragmented following the procedure presented in Ref. [2]. For a given temperature, the fragments produced in all realizations were collected in a single mass histogram of logarithmic bins, normalized to generate the distribution $n(m)$, and further integrated to obtain the cumulative form $F(m)$.

In Fig. 1 we show the log-log plot of $F(m)$ for $T^*=0.37$. It is clear from this figure that $F(m)$ displays power-law behavior for intermediate mass values. Moreover, this is followed by a sudden cutoff that decays faster than exponential. An entirely similar behavior for impact fragmentation was observed experimentally by Meibom and Balslev [3] and through numerical simulations by Inaoka *et al.* [4]. Based on these features of $F(m)$, we used the following empirical expression to fit our simulation results [5]:

$$F(m) \sim m^{-\alpha} \exp[-(m/M_0)^\gamma], \tag{2}$$

where M_0 and γ are cutoff parameters. The solid line in Fig. 1 is the best fit we found for the data with $\alpha=0.082$, $M_0=1700$, and $\gamma=2$. Although close to zero, this value of α is sufficiently large to characterize the power-law signature of the cumulative mass distribution. At this point, it is important to show that the scaling ansatz (2) is also consistent with the

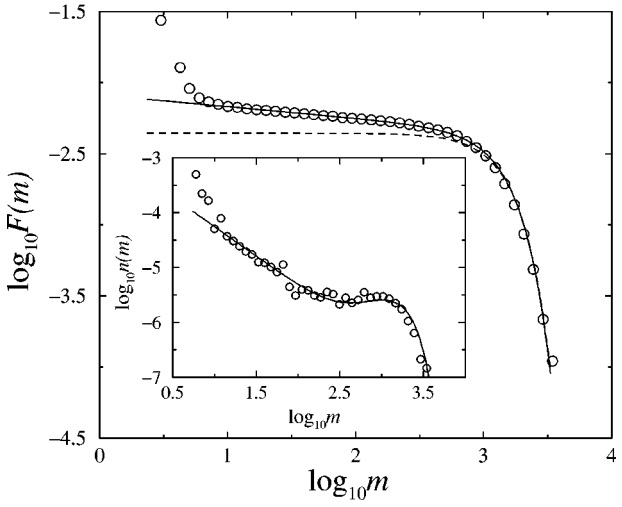

FIG. 1. Log-log plot of the cumulative mass distribution of fragments $F(m)$ for $T^*=0.37$ and $R=0.43$ (circles). Here R is a measure of the input energy as described in Ref. [2]. The solid line is the best fit-function to the data for $m>10$, $F(m) \sim m^{-0.082} \exp[-(m/1700)^2]$. For comparison, the dashed line corresponds to $f(m) \sim \exp[-(m/1700)^2]$. The inset shows the corresponding mass distribution $n(m)$ (circles), with the solid line being the best fit [Eq. (3)] to the simulation data.

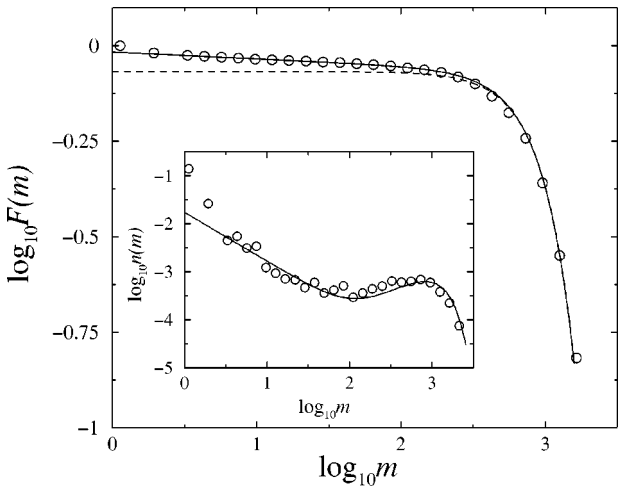

FIG. 2. Same as in Fig. 1, but for $T^*=0.037$ (circles). The solid line is the best fit to the data for $m>2$, $F(m) \sim m^{-0.018} \exp[-(m/1200)^2]$. The dashed line corresponds to $f(m) \sim \exp[-(m/1200)^2]$. In the inset we show the mass distribution $n(m)$ (circles), with the best fit [Eq. (3), solid line] to the simulation data.

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observed behavior for $n(m)$. Accordingly, the corresponding scaling function for the fragment mass distribution $n(m)$ can be written as

$$n(m) \sim m^{-(\alpha+1)} f(m/M_0), \tag{3}$$

where $f(z)=(\alpha+\gamma z^{\gamma})\exp(-z^{\gamma})$. The inset of Fig. 1 shows that Eq. (3), with the same values of the parameters obtained for $F(m)$, fits very well the simulation data for $n(m)$ over the whole range of relevant fragment masses. In Ref. [2], the form and the parameters of $f(z)$ have not been determined due to the limited number of fragmentation samples available.

The same approach has been applied with success to a lower temperature, $T^*=0.037$. As shown in Fig. 2, although the simulated data present more fluctuations due to the poorer statistics of intermediate mass fragments (see the inset in Fig. 2), it is still possible to fit the cumulative mass distribution $F(m)$ using the same scaling expression Eq. (2), but now with $\alpha=0.018$, $M_0=1200$, and $\gamma=2$. Once more, the behavior of $n(m)$ is essentially compatible with the expected scaling from Eq. (3) using the fitting parameters of $F(m)$. As a matter of fact, the results shown here are in better agreement with the hypotheses of $1/m$ scaling for $n(m)$ than the results presented in Ref. [2]. This behavior is related to the fact that we are now using a larger number of samples and an improved technique to analyze the fragmentation data.

Concluding, we agree with the point raised in the Comment by Åström *et al.* [1] that the analysis for small fragment masses in terms of $N(m)$ may be misleading. In spite of this fact, we could confirm here that our computational data on the fragmentation of two-dimensional thermalized objects clearly possess a region of intermediate fragment masses following a power-law behavior, with an exponent close to -1. Under these circumstances, we strongly believe that, although correct, the above-mentioned Comment [1] does not invalidate the main result of our previous work [2].

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